1.(1) $A^k = \begin{bmatrix} x^k & 0 \\ -2x^k+2y^k & y^k \end{bmatrix}$;
(2) $-\frac{16}{3}$;
(3) $C_{23} = -3, C_{33} = 0, \det A = 6$;
(4) 7;
(5) $A = \begin{bmatrix} 2s & 3 \\ 1 & -s \end{bmatrix}$, $\det A_2(b) = 2s^2 - 1$
(6) constant, 1, ∞;
(7) $\begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$;
(8) 0;
(9) $x_1 = \begin{bmatrix} \frac{5}{2} \\ \frac{3}{2} \end{bmatrix}$;
(10) the number of eigenvalues is 2, the eigenvalues with algebraic multiplicity 2 are 1 and 2.

2. The major steps are the following.

Step1: $\det(A - xI) = \begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1-x \end{vmatrix} = x^2(3 - x)$.

Step2: Find eigenvalues and eigenvectors. The eigenvalues are 3 and 0. The eigenvector for $\lambda = 3$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. A basis for the eigenspace corresponding to $\lambda = 0$ is $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

Step3: Write down D and P. $D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

3.(a) $T(p) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; (b) The solution is not unique. A simplest solution is $p(t) = t$; (c) The matrix for T is $\begin{bmatrix} -1 & -1 & 1 \\ -1 & 0 & 0 \end{bmatrix}$.

4. $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ is always diagonalisable. The reason is that this matrix has two distinct eigenvalues $a \pm b$ when $b \neq 0$. And when $b = 0$ the matrix is already diagonal.

$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ is never diagonalisable by conjugating real matrices except when $b = 0$. The reason is that this matrix has no real eigenvalue when $b \neq 0$.

5. The major steps are:

Step (1): Performing elementary row operations. We reach the row echelon form $\begin{bmatrix} 1 & 2 & 1 & -1 & 2 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and the reduced echelon form $\begin{bmatrix} 1 & 0 & 1 & 0 & -8 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Step (2): Using the reduced echelon form to write down the basis for $\mathrm{Col}A, \mathrm{Nul}A, \mathrm{Row}A$.

A basis for $\mathrm{Col}A$ is $\begin{bmatrix} 2 \\ -1 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \\ 15 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \\ 2 \end{bmatrix}$.

A basis for $\mathrm{Nul}A$ is $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ -4 \\ 0 \\ 2 \\ 1 \end{bmatrix}$.

A basis for $\mathrm{Row}A$ is $(1, 0, 1, 0, -8), (0, 1, 0, 0, 4), (0, 0, 0, 1, -2)$.

6. First computing the characteristic polynomial of $A = \begin{bmatrix} \frac{3}{4} & p \\ \frac{1}{16} & \frac{3}{4} \end{bmatrix}$. $\det(A - \lambda I) = (\frac{3}{4} - \lambda)^2 - \frac{1}{16}p$.

Case (a): The eigenvalues are real. So $\lambda = \frac{3}{4} \pm \frac{1}{4}\sqrt{p}$.

Condition for attractor: $0 < \frac{3}{4} \pm \frac{1}{4}\sqrt{p} < 1$. This implies that $0 < p < 1$.

Condition for saddle point: $0 < \frac{3}{4} - \frac{1}{4}\sqrt{p} < 1$ and $1 < \frac{3}{4} + \frac{1}{4}\sqrt{p}$. This implies that $1 < p < 9$.

Case (b): when $-1 < p < 0$, we have $\lambda = \frac{3}{4} \pm \frac{i}{4}\sqrt{p}$. We see that $|\lambda| < 1$. Therefore the origin is an attractor.

Case (c): The two eigenvalues are 1 and $\frac{1}{2}$. The corresponding eigenvectors are $v_1 = \left[\begin{smallmatrix}4\\1\end{smallmatrix}\right]$ and $v_2 = \left[\begin{smallmatrix}-4\\1\end{smallmatrix}\right]$. The general solution for the discrete dynamic system is $x_k = c_1\left[\begin{smallmatrix}4\\1\end{smallmatrix}\right] + c_2(\frac{1}{2})^k\left[\begin{smallmatrix}-4\\1\end{smallmatrix}\right]$.

Case (d): The equation for l_2 is $y = \frac{1}{4}x$. The parametric equation for l_1 is $\left[\begin{smallmatrix}r_1\\r_2\end{smallmatrix}\right] + t\left[\begin{smallmatrix}-4\\1\end{smallmatrix}\right]$.